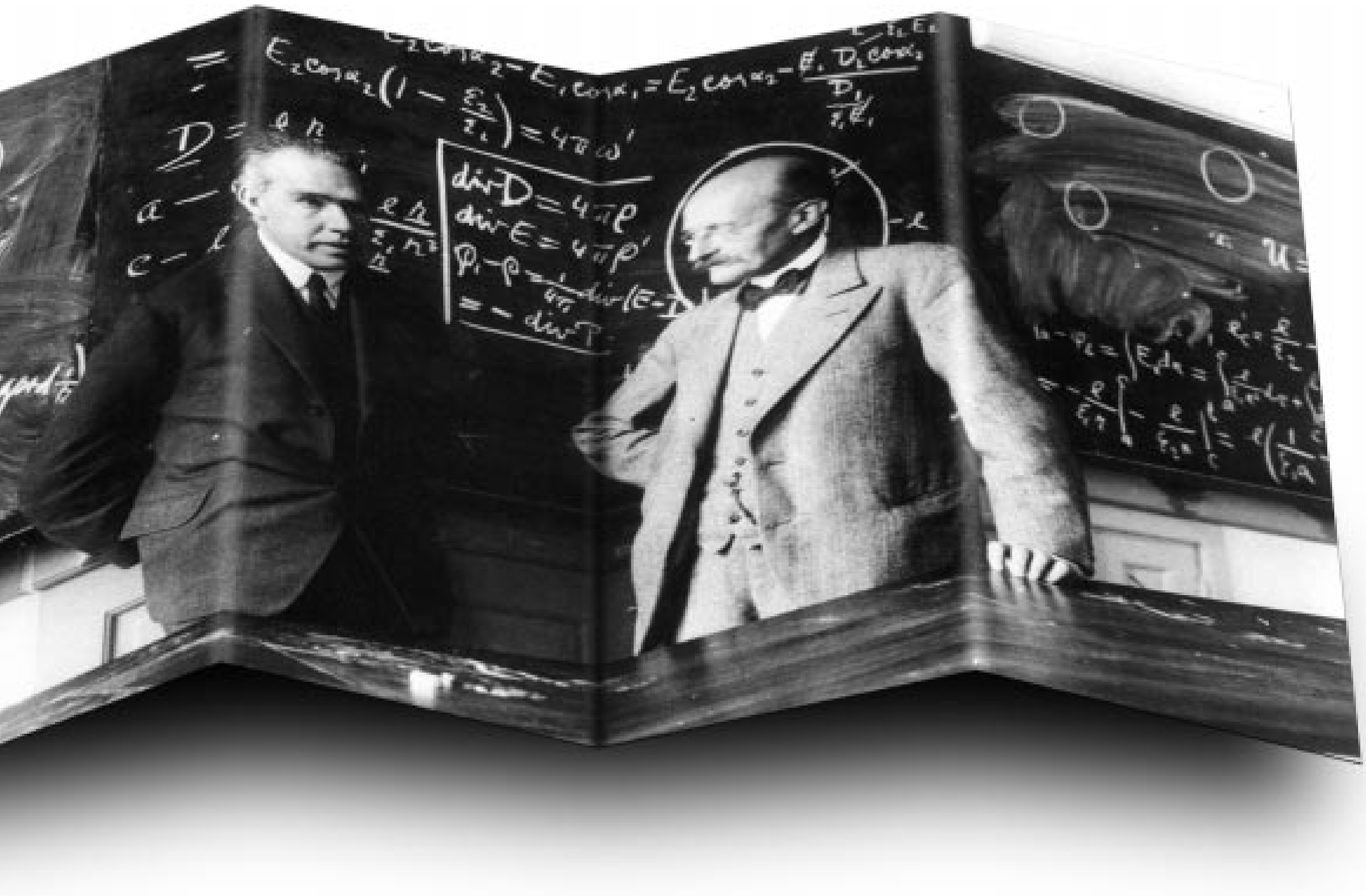
THE ORIGINS OF THE QUANTUM THEORY

by CATHRYN CARSON

WHAT IS A QUANTUM THEORY? We have been asking that question for a long time, ever since Max Planck introduced the element of discontinuity we call the quantum a century ago. Since then, the chunkiness of Nature (or at least of our theories about it) has been built into our basic conception of the world. It has prompted a fundamental rethinking of physical theory. At the same time it has helped make sense of a whole range of peculiar behaviors manifested principally at microscopic levels.

From its beginning, the new regime was symbolized by Planck's constant *h*, introduced in his famous paper of 1900. Measuring the world's departure from smooth, continuous behavior, *h* proved to be a very small number, but different from zero. Wherever it appeared, strange phenomena came with it. What it really meant was of course mysterious. While the quantum era was inaugurated in 1900, a quantum theory would take much longer to jell. Introducing discontinuity was a tentative step, and only a first one. And even thereafter, the recasting of physical theory was hesitant and slow. Physicists pondered for years what a quantum theory might be. Wondering how to integrate it with the powerful apparatus of nineteenth-century physics, they also asked what relation it bore to existing, "classical" theories. For some the answers crystallized with quantum mechanics, the result of a quarter-century's labor. Others held out for further rethinking. If the outcome was not to the satisfaction of all, still the quantum theory proved remarkably

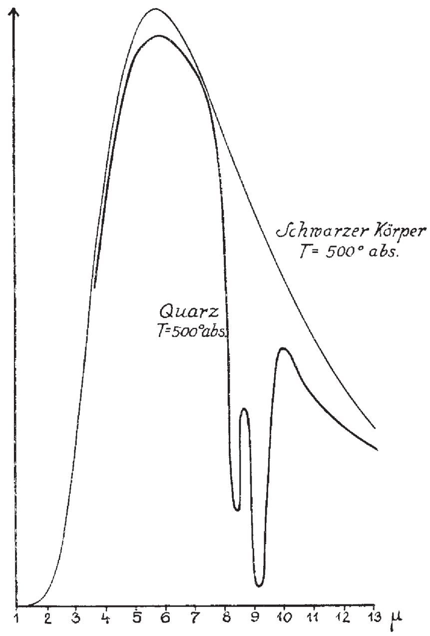


Two theorists, Niels Bohr and Max Planck, at the blackboard. (Courtesy Emilio Segrè Visual Archives, Margrethe Bohr Collection)

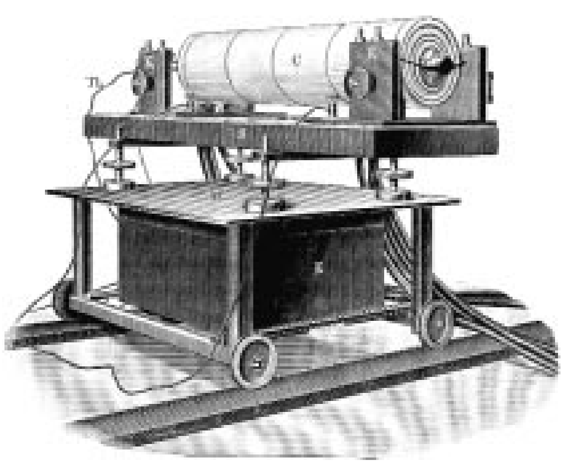
successful, and the puzzlement along the way, despite its frustrations, can only be called extraordinarily productive.

INTRODUCING *h*

The story began inconspicuously enough on December 14, 1900. Max Planck was giving a talk to the German Physical Society on the continuous spectrum of the frequencies of light emitted by an ideal heated body. Some two months earlier this 42-year-old theorist had presented a formula capturing some new experimental results. Now, with leisure to think and more time at his disposal, he sought to provide a physical justification for his formula. Planck pictured a piece of matter, idealizing it somewhat, as equivalent to a collection of oscillating electric charges. He then imagined distributing its energy in discrete chunks proportional to the frequencies of oscillation. The constant of proportionality he chose to call h; we would now write $\varepsilon = hf$. The frequencies of oscillation determined the frequencies of the emitted light. A twisted chain of reasoning then reproduced Planck's postulated formula, which now involved the same natural constant h.



An ideal blackbody spectrum (Schwarzer Körper) and its real-world approximation (quartz). (From Clemens Schaefer, Einführung in die Theoretische Physik, 1932).



Experimental setup for measuring blackbody radiation. (The blackbody is the tube labeled C.) This design was a product of Germany's Imperial Institute of Physics and Technology in Berlin, where studies of blackbodies were pursued with an eye toward industrial standards of luminous intensity. (From Müller-Pouillets Lehrbuch der Physik, 1929).

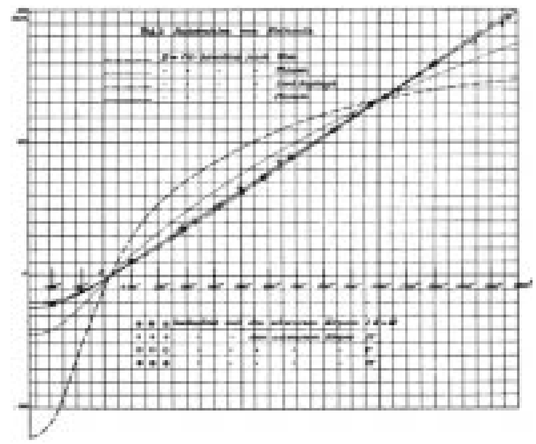
Looking back on the event, we might expect revolutionary fanfare. But as so often in history, matters were more ambiguous. Planck did not call his energy elements quanta and was not inclined to stress their discreteness, which made little sense in any familiar terms. So the meaning of his procedure only gradually became apparent. Although the problem he was treating was pivotal in its day, its implications were at first thought to be confined.

BLACKBODIES

The behavior of light in its interaction with matter was indeed a key problem of nineteenth-century physics. Planck was interested in the two theories that overlapped in this domain. The first was electrodynamics, the theory of electricity, magnetism, and light waves, brought to final form by James Clerk Maxwell in the 1870s. The second, dating from roughly the same period, was thermodynamics and statistical mechanics, governing transformations of energy and its behavior in time. A pressing question was whether these two grand theories could be fused into one, since they started from different fundamental notions.

Beginning in the mid-1890s, Planck took up a seemingly narrow problem, the interaction of an oscillating charge with its electromagnetic field. These studies, however, brought him into contact with a long tradition of work on the emission of light. Decades earlier it had been recognized that perfectly absorbing ("black") bodies provided a standard for emission as well. Then over the years a small industry had grown up around the study of such objects (and their real-world substitutes, like soot). A small group of theorists occupied themselves with the thermodynamics of radiation, while a host of experimenters labored over heated bodies to fix temperature, determine intensity, and characterize deviations from blackbody ideality (see the graph above). After scientists pushed the practical realization of an old idea—that a closed tube with a small hole constituted a near-ideal blackbody—this "cavity radiation" allowed ever more reliable measurements. (See illustration at left.)

Now Planck's oscillating charges emitted and absorbed radiation, so they could be used to model a blackbody. Thus everything seemed to fall into place in 1899 when he reproduced a formula that a colleague had derived by less secure means. That was convenient; everyone agreed that Willy Wien's formula matched the observations. The trouble was that immediately afterwards, experimenters began finding deviations. At low frequencies, Wien's expression became increasingly untenable, while elsewhere it continued to work well enough. Informed of the results in the fall of 1900, on short notice Planck came up with a reasonable interpolation. With its adjustable constants his formula seemed to fit the experiments (see graph at right). Now the question became: Where might it come from? What was its physical meaning?

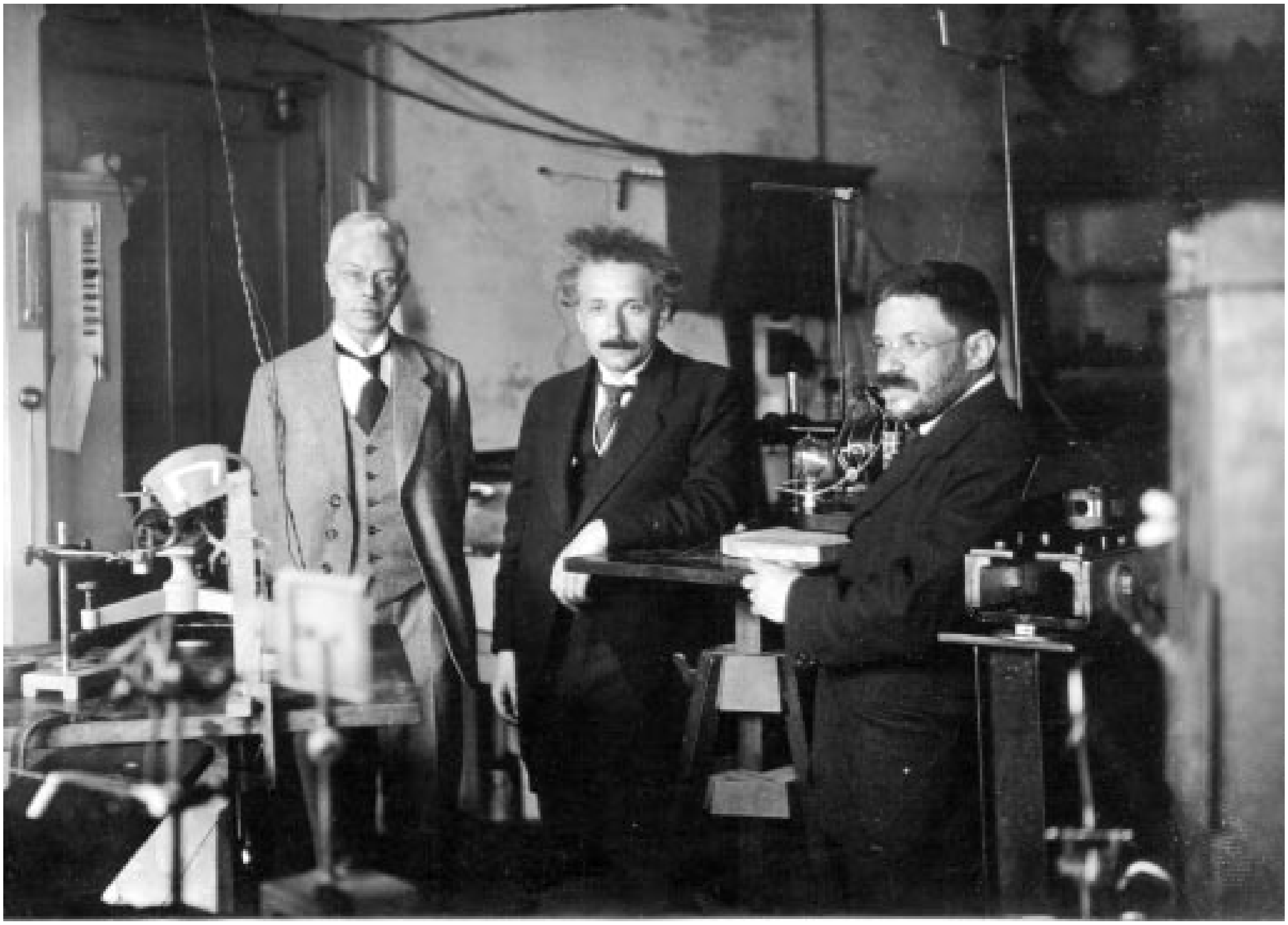


Experimental and theoretical results on the blackbody spectrum. The data points are experimental values; Planck's formula is the solid line. (Reprinted from H. Rubens and F. Kurlbaum, Annalen der Physik, *1901.)*

As we saw, Planck managed to produce a derivation. To get the right statistical results, however, he had to act as though the energy involved were divided up into elements $\varepsilon = hf$. The derivation was a success and splendidly reproduced the experimental data. Its meaning was less clear. After all, Maxwell's theory already gave a beautiful account of light—and treated it as a wave traveling in a continuous medium. Planck did not take the constant h to indicate a physical discontinuity, a real atomicity of energy in a substantive sense. None of his colleagues made much of this possibility, either, until Albert Einstein took it up five years later.

MAKING LIGHT QUANTA REAL

Of Einstein's three great papers of 1905, the one "On a Heuristic Point of View Concerning the Production and Transformation of Light" was the piece that the 26-year-old patent clerk labeled revolutionary. It was peculiar, he noted, that the electromagnetic theory of light assumed a continuum, while current accounts of matter started from discrete atoms. Could discontinuity be productive for light as well? However indispensable Maxwell's equations might seem, for some interesting phenomena they proved inadequate. A key example was blackbody radiation, which Einstein now looked at in a way different from Planck. Here a rigorously classical treatment, he showed,



Pieter Zeeman, Albert Einstein, and Paul Ehrenfest (left to right) in Zeeman's Amsterdam laboratory. (Courtesy Emilio Segrè Visual Archives, W. F. Meggers Collection)

yielded a result not only wrong but also absurd. Even where Wien's law was approximately right (and Planck's modification unnecessary), elementary thermodynamics forced light to behave as though it were localized in discrete chunks. Radiation had to be parcelled into what Einstein called "energy quanta." Today we would write $E = hf$.

Discontinuity was thus endemic to the electromagnetic world. Interestingly, Einstein did not refer to Planck's constant h, believing his approach to be different in spirit. Where Planck had looked at oscillating charges, Einstein applied thermodynamics to the light itself. It was only later that Einstein went back and showed how Planck's work implied real quanta. In the meantime, he offered a further, radical extension. If light behaves on its own as though composed of such quanta, then perhaps it is also emitted and absorbed in that fashion. A few easy considerations then yielded a law for the photoelectric effect, in which light ejects electrons from the surface of a metal. Einstein provided not only a testable hypothesis but also a new way of measuring the constant h (see table on the next page).

Today the photoelectric effect can be checked in a college laboratory. In 1905, however, it was far from trivial. So it would remain for more than a decade. Even after Robert Millikan confirmed Einstein's prediction, he and others balked at the underlying quantum hypothesis. It still violated everything known about light's wavelike behavior (notably, interference) and hardly seemed reconcilable with Maxwell's equations. When Einstein was awarded the Nobel Prize, he owed the honor largely to the photoelectric effect. But the citation specifically noted his discovery of the law, not the explanation that he proposed.

The relation of the quantum to the wave theory of light would remain a point of puzzlement. Over the next years Einstein would only sharpen the contradiction. As he showed, thermodynamics ineluctably required both classical waves and quantization. The two aspects were coupled: both were necessary, and at the same time. In the process, Einstein moved even closer to attributing to light a whole panoply of particulate properties. The particle-like quantum, later named the photon, would prove suggestive for explaining things like the scattering of X rays. For that 1923 discovery, Arthur Compton would win the Nobel Prize. But there we get ahead of the story. Before notions of wave-particle duality could be taken seriously, discontinuity had to demonstrate its worth elsewhere.

BIRGE'S CALCULATION OF PLANCK'S UNIVERSAL CONSTANT h BY VARIOUS METHODS

Value of h.	Method.	Remarks.
6·551 ± 0·009	$\sigma = 5{\cdot}72$	Total radiation
6·557 ± 0·013	$C_2 = 14330$	Spectral radiation
6·542 ± 0·011	Rydberg constant	Bohr's theory of atomic structure
6·578 ± 0·026	Photo-electric equations	Einstein's equation
6·555 ± 0·009	X-rays	Quantum relation
6·560 ± 0·014	Ultimate rational units	Theory of Lewis and Adams
6·579 ± 0·021	Ionisation potential	..

Mean value, $h = (6{\cdot}5543 \pm 0{\cdot}0025)10^{-27}$ erg sec.

(From W. W. Coblentz, Radiation, Determination of the Constants and Verification of the Laws in A Dictionary of Applied Physics, Vol. 4, Ed. Sir Richard Glazebrook, 1923)

BEYOND LIGHT

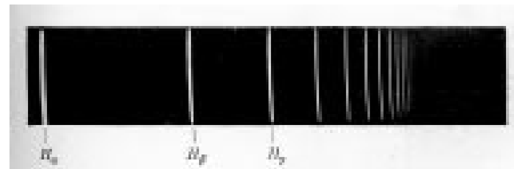
As it turned out, the earliest welcome given to the new quantum concepts came in fields far removed from the troubled theories of radiation. The first of these domains, though hardly the most obvious, was the theory of specific heats. The specific heat of a substance determines how much of its energy changes when its temperature is raised. At low temperatures, solids display peculiar behavior. Here Einstein suspected—again we meet Einstein—that the deviance might be explicable on quantum grounds. So he reformulated Planck's problem to handle a lattice of independently vibrating atoms. From this highly simplistic model, he obtained quite reasonable predictions that involved the same quantity hf, now translated into the solid-state context.

There things stood for another three years. It took the sudden attention of the physical chemist Walther Nernst to bring quantum theories of specific heats to general significance. Feeling his way towards a new law of thermodynamics, Nernst not only bolstered Einstein's ideas with experimental results, but also put them on the agenda for widespread discussion. It was no accident, and to a large degree Nernst's doing, that the first Solvay Congress in 1911 dealt precisely with radiation theory and quanta (see photograph below). Einstein spoke on specific heats, offering additional comments on electromagnetic radiation. If the quantum was born in 1900, the Solvay meeting was, so to speak, its social debut.

What only just began to show up in the Solvay colloquy was the other main realm in which discontinuity would prove its value. The technique of quantizing oscillations applied, of course, to line spectra as well. In contrast to the universality of blackbody radiation, the discrete lines of light emission and absorption varied immensely from one substance to the next. But the regularities evident even in the welter of the lines provided fertile matter for quantum conjectures. Molecular spectra turned into an all-important site of research during

The first Solvay Congress in 1911 assembled the pioneers of quantum theory. Seated (left to right): W. Nernst, M. Brillouin, E. Solvay, H. A. Lorentz, E. Warburg, J. Perrin, W. Wien, M. Curie, H. Poincaré. Standing (left to right): R. Goldschmidt, M. Planck, H. Rubens, A. Sommerfeld, F. Lindemann, M. de Broglie, M. Knudsen, F. Hasenöhrl, G. Hostelet, E. Herzen, J. Jeans, E. Rutherford, H. Kamerlingh Onnes, A. Einstein, P. Langevin. (From Cinquantenaire du Premier Conseil de Physique Solvay, 1911–1961).

the quantum's second decade. Slower to take off, but ultimately even more productive, was the quantization of motions within the atom itself. Since no one had much sense of the atom's constitution, the venture into atomic spectra was allied to speculative model-building. Unsurprisingly, most of the guesses of the early 1910s turned out to be wrong. They nonetheless sounded out the possibilities. The orbital energy of electrons, their angular momentum (something like rotational inertia), or the frequency of their small oscillations about equilibrium: all these were fair game for quantization. The observed lines of the discrete spectrum could then be directly read off from the electrons' motions.



The line spectrum of hydrogen. (From G. Herzberg, Annalen der Physik, *1927)*

THE BOHR MODEL OF THE ATOM

It might seem ironic that Niels Bohr initially had no interest in spectra. He came to atomic structure indirectly. Writing his doctoral thesis on the electron theory of metals, Bohr had become fascinated by its failures and instabilities. He thought they suggested a new type of stabilizing force, one fundamentally different from those familiar in classical physics. Suspecting the quantum was somehow implicated, he could not figure out how to work it into the theory.

The intuition remained with him, however, as he transferred his postdoctoral attention from metals to Rutherford's atom. When it got started, the nuclear atom (its dense positive center circled by electrons) was simply one of several models on offer. Bohr began working on it during downtime in Rutherford's lab, thinking he could improve on its treatment of scattering. When he noticed that it ought to be unstable, however, his attention was captured for good. To stabilize the model by fiat, he set about imposing a quantum condition, according to the standard practice of the day. Only after a colleague directed his attention to spectra did he begin to think about their significance.

The famous Balmer series of hydrogen was manifestly news to Bohr. (See illustration above.) He soon realized, however, that he could fit it to his model—if he changed his model a bit. He reconceptualized light emission as a transition between discontinuous orbits, with the emitted frequency determined by $\Delta E = hf$. To get the

What Was Bohr Up To?

BOHR'S PATH to his atomic model was highly indirect. The rules of the game, as played in 1911–1912, left some flexibility in what quantum condition one imposed. Initially Bohr applied it to multielectron states, whose allowed energies he now broke up into a discrete spectrum. More specifically, he picked out their orbital frequencies to quantize. This is exactly what he would later proscribe in the final version of his model.

He rethought, however, in the face of the Balmer series and its simple numerical pattern

$$\mathbf{f_n} = \mathbf{R}\,(1/4 - 1/\mathbf{n}^2).$$

Refocusing on one electron and highlighting excited states, he reconceptualized light emission as a transition. The Balmer series then resulted from a tumble from orbit **n** to orbit 2; the Rydberg constant **R** could be determined in terms of **h**. However, remnants of the earlier model still appeared in his paper.

orbits' energies right, Bohr had to introduce some rather ad hoc rules. These he eventually justified by quantization of angular momentum, which now came in units of Planck's constant h. (He also used an interesting asymptotic argument that will resurface later.)

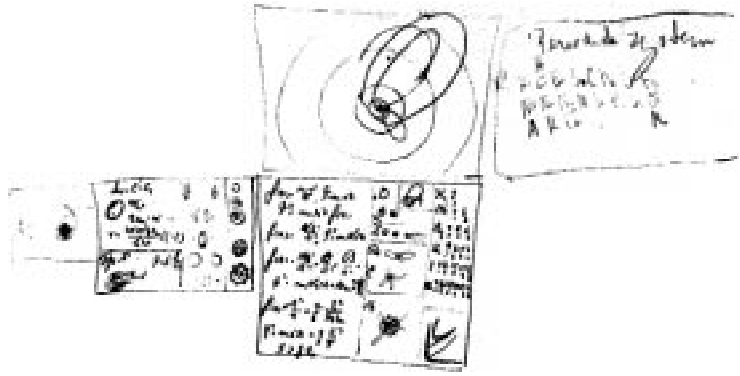
Published in 1913, the resulting picture of the atom was rather odd. Not only did a quantum condition describe transitions between levels, but the "stationary states," too, were fixed by nonclassical fiat. Electrons certainly revolved in orbits, but their frequency of revolution had nothing to do with the emitted light. Indeed, their oscillations were postulated *not* to produce radiation. There was no predicting when they might jump between levels. And transitions generated frequencies according to a quantum relation, but Bohr proved hesitant to accept anything like a photon.

The model, understandably, was not terribly persuasive—that is, until new experimental results began coming in on X rays, energy levels, and spectra. What really convinced the skeptics was a small modification Bohr made. Because the nucleus is not held fixed in space, its mass enters in a small way into the spectral frequencies. The calculations produced a prediction that fit to 3 parts in 100,000—pretty good, even for those days when so many numerical coincidences proved to be misleading.

The wheel made its final turn when Einstein connected the Bohr atom back to blackbody radiation. His famous papers on radiative transitions, so important for the laser (see following article by Charles Townes), showed the link among Planck's blackbody law, discrete energy levels, and quantized emission and absorption of radiation. Einstein further stressed that transitions could not be predicted in anything more than a probabilistic sense. It was in these same papers, by the way, that he formalized the notion of particle-like quanta.

THE OLD QUANTUM THEORY

What Bohr's model provided, like Einstein's account of specific heats, was a way to embed the quantum in a more general theory. In fact, the study of atomic structure would engender something plausibly called a quantum theory, one that began reaching towards a full-scale replacement for classical physics. The relation between the old and



Above: Bohr's lecture notes on atomic physics, 1921. (Original source: Niels Bohr Institute, Copenhagen. From Owen Gingerich, Ed., Album of Science: The Physical Sciences in the Twentieth Century*, 1989.)*

Left: Wilhelm Oseen, Niels Bohr, James Franck, Oskar Klein (left to right), and Max Born, seated, at the Bohr Festival in Göttingen, 1922. (Courtesy Emilio Segrè Visual Archives, Archive for the History of Quantum Physics)

the new became a key issue. For some features of Bohr's model preserved classical theories, while others presupposed their breakdown. Was this consistent or not? And why did it work?

By the late 1910s physicists had refined Bohr's model, providing a relativistic treatment of the electrons and introducing additional "quantum numbers." The simple quantization condition on angular momentum could be broadly generalized. Then the techniques of nineteenth-century celestial mechanics provided powerful theoretical tools. Pushed forward by ingenious experimenters, spectroscopic studies provided ever more and finer data, not simply on the basic line spectra, but on their modulation by electric and magnetic fields. And abetted by their elders, Bohr, Arnold Sommerfeld, and Max Born, a generation of youthful atomic theorists cut their teeth on such problems. The pupils, including Hendrik Kramers, Wolfgang Pauli, Werner Heisenberg, and Pascual Jordan, practiced a tightrope kind of theorizing. Facing resistant experimental data, they balanced the empirical evidence from the spectra against the ambiguities of the prescriptions for applying the quantum rules.

Within this increasingly dense body of work, an interesting strategy began to jell. In treating any physical system, the first task was to identify the possible classical motions. Then atop these classical motions, quantum conditions would be imposed. Quantization became a regular procedure, making continual if puzzling reference back to classical results. In another way, too, classical physics served as a touchstone. Bohr's famous "correspondence principle"

Paul Dirac and Werner Heisenberg in Cambridge, circa 1930. (Courtesy Emilio Segrè Visual Archives, Physics Today Collection)

first found application in his early papers, where it provided a deeper justification of his quantization condition. In the "classical limit," for large orbits with high quantum numbers and small differences in energy, the radiation from transitions between adjacent orbits should correspond to the classical radiation frequency. Quantum and classical results must match up. Employed with a certain Bohrian discrimination, the correspondence principle yielded detailed information about spectra. It also helped answer the question: How do we build up a true quantum theory?

Not that the solution was yet in sight. The old quantum theory's growing sophistication made its failures ever plainer. By the early 1920s the theorists found themselves increasingly in difficulties. No single problem was fatal, but their accumulation was daunting. This feeling of crisis was not entirely unspecific. A group of atomic theorists, centered on Bohr, Born, Pauli, and Heisenberg, had come to suspect that the problems went back to electron trajectories. Perhaps it was possible to abstract from the orbits? Instead they focused on transition probabilities and emitted radiation, since those might be more reliably knowable. "At the moment," Pauli still remarked in the spring of 1925, "physics is again very muddled; in any case, it is far too difficult for me, and I wish I were a movie comedian or something of the sort and had never heard of physics."

QUANTUM MECHANICS

Discontinuity, abstraction from the visualizable, a positivistic turn towards observable quantities: these preferences indicated one path to a full quantum theory. So, however, did their opposites. When in 1925–1926 a true quantum mechanics was finally achieved, two seemingly distinct alternatives were on offer. Both took as a leitmotif the relation to classical physics, but they offered different ways of working out the theme.

Heisenberg's famous paper of 1925, celebrated for launching the transformations to come, bore the title "On the Quantum-Theoretical Reinterpretation of Kinematical and Mechanical Relations." The point of departure was Bohr's tradition of atomic structure: discrete states remained fundamental, but now dissociated from intuitive representation. The transformative intent was even broader from the start. Heisenberg translated classical notions into quantum ones in the best correspondence-principle style. In his new quantum mechanics, familiar quantities behaved strangely; multiplication depended on the order of the terms. The deviations, however, were calibrated by Planck's constant, thus gauging the departure from classical normality.

Some greeted the new theory with elation; others found it unsatisfactory and disagreeable. For as elegant as Heisenberg's translation might be, it took a very partial view of the problems of the quantum. Instead of the quantum theory of atomic structure, one might also start from the wave-particle duality. Here another young theorist, Louis de Broglie, had advanced a speculative proposal in 1923 that Heisenberg and his colleagues had made little of. Thinking over the discontinuous, particle-like aspects of light, de Broglie suggested looking for continuous, wave-like aspects of electrons. His notion, while as yet ungrounded in experimental evidence, did provide surprising insight into the quantum conditions for Bohr's orbits. It also, by a sideways route, gave Erwin Schrödinger the idea for his brilliant papers of 1926.

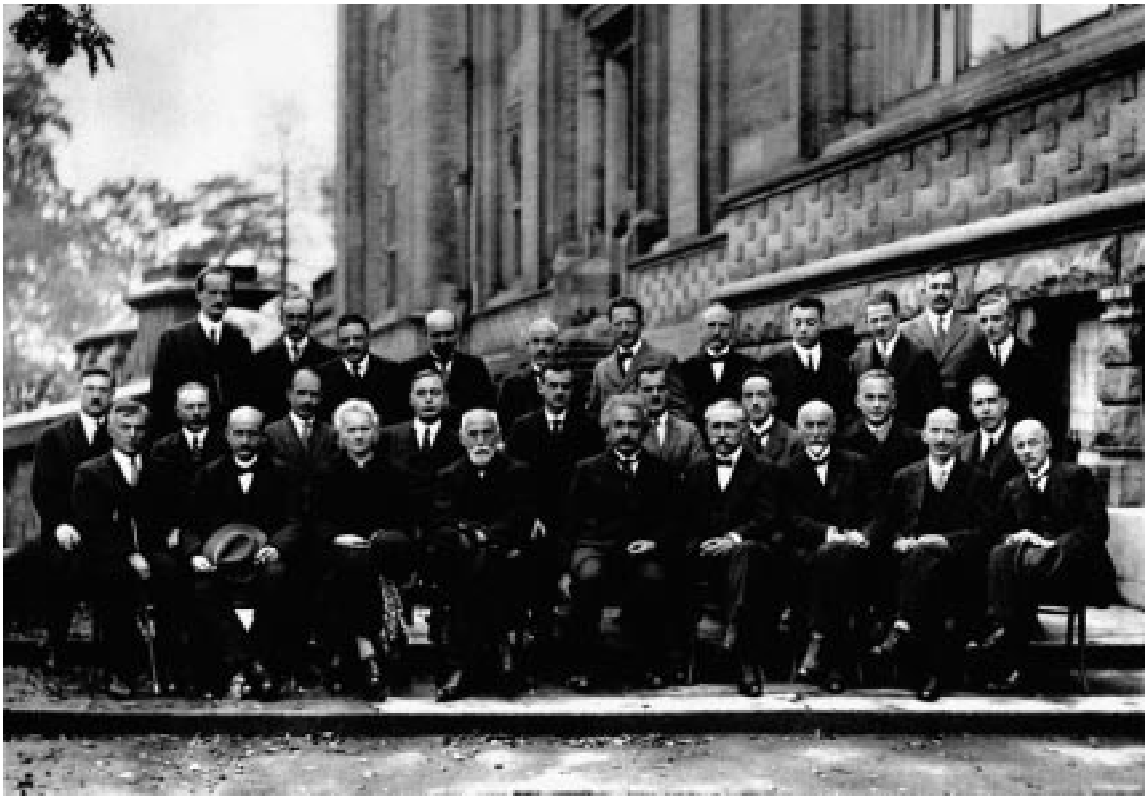
Imagining the discrete allowed states of any system of particles as simply the stable forms of continuous matter waves, Schrödinger sought connections to a well-developed branch of classical physics. The techniques of continuum mechanics allowed him to formulate an equation for his waves. It too was built around the constant h. But now the basic concepts were different, and so also the fundamental meaning. Schrödinger had a distaste for the discontinuity of Bohr's atomic models and the lack of intuitive picturability of Heisenberg's quantum mechanics. To his mind, the quantum did not imply any of these things. Indeed, it showed the opposite: that the apparent atomicity of matter disguised an underlying continuum.

The Quantum in Quantum Mechanics

ICONICALLY we now write Heisenberg's relations as

$$\mathbf{pq} - \mathbf{qp} = -\,i\mathbf{h}/2\pi.$$

Here **p** represents momentum and **q** represents position. For ordinary numbers, of course, **pq** equals **qp**, and so **pq** – **qp** is equal to zero. In quantum mechanics, this difference, called the commutator, is now measured by **h**. (The same thing happens with Heisenberg's uncertainty principle of 1927: $\Delta\mathbf{p}\Delta\mathbf{q} \geq \mathbf{h}/4\pi$.) The significance of the approach, and its rearticulation as a matrix calculus, was made plain by Max Born, Pascual Jordan, and Werner Heisenberg. Its full profundity was revealed by Paul Dirac.



Old faces and new at the 1927 Solvay Congress. The middle of the second row lines up Hendrik Kramers, Paul Dirac, Arthur Compton, and Louis de Broglie. Behind Compton stands Erwin Schrödinger, with Wolfgang Pauli and Werner Heisenberg next to each other behind Max Born. (From Cinquantenaire du Premier Conseil de Physique Solvay, 1911–1961)

Thus a familiar model connected to physical intuition, but constituting matter of some ill-understood sort of wave, confronted an abstract mathematics with seemingly bizarre variables, insistent about discontinuity and suspending space-time pictures. Unsurprisingly, the coexistence of alternative theories generated debate. The fact, soon demonstrated, of their mathematical equivalence did not resolve the interpretative dispute. For fundamentally different physical pictures were on offer.

In fact, in place of Schrödinger's matter waves and Heisenberg's uncompromising discreteness, a conventional understanding settled in that somewhat split the difference. However, the thinking of the old quantum theory school still dominated. Born dematerialized Schrödinger's waves, turning them into pure densities of probability for finding discrete particles. Heisenberg added his uncertainty principle, limiting the very possibility of measurement and undermining the law of causality. The picture was capped by Bohr's notion of complementarity, which sought to reconcile contradictory concepts like waves and particles.

Labeled the Copenhagen Interpretation after Bohr's decisive influence, its success (to his mind) led the Danish theorist to characterize quantum mechanics as a rational generalization of classical physics. Not everyone agreed that this was the end point. Indeed, Einstein, Schrödinger, and others were never reconciled. Even Bohr, Heisenberg, and Pauli expected further changes—though in a new domain, the quantum theory of fields, which took quantum mechanics to a higher degree of complexity. But their expectations of fundamental transformation in the 1930s and beyond, characterized by analogies to the old quantum theory, found little resonance outside of their circle.

Ironically enough, just as for their anti-Copenhagen colleagues, their demand for further rethinking did not make much headway. If the physical meaning of the quantum remained, to some, rather obscure, its practical utility could not be denied. Whatever lessons one took from quantum mechanics, it seemed to work. It not only incorporated gracefully the previous quantum phenomena, but opened the door to all sorts of new applications. Perhaps this kind of success was all one could ask for? In that sense, then, a quarter-century after Planck, the quantum had been built into the foundations of physical theory.

SUGGESTIONS FOR FURTHER READING

Olivier Darrigol, **From c-Numbers to q-Numbers: The Classical Analogy in the History of Quantum Theory** (Berkeley: U.C. Press, 1992).

Helge Kragh, **Quantum Generations: A History of Physics in the Twentieth Century** (Princeton, Princeton University Press, 1999).

Abraham Pais, **"Subtle is the Lord ...": The Science and the Life of Albert Einstein** (Oxford, Oxford University Press, 1982).

Helmut Rechenberg, "Quanta and Quantum Mechanics," in Laurie M. Brown, Abraham Pais, and Sir Brian Pippard, Eds., **Twentieth Century Physics**, Vol. 1 (Bristol and New York, Institute of Physics Publishing and American Institute of Physics Press, 1995), pp. 143-248.

The August 11, 2000, issue of **Science** (Vol. 289) contains an article by Daniel Kleppner and Roman Jackiw on "One Hundred Years of Quantum Physics." To see the full text of this article, go to http://www.sciencemag.org/cgi/content/full/289/5481/893.